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EFFICIENCY ASPECTS OF SHORT LOW VELOCITY RAILGUNS(U)  
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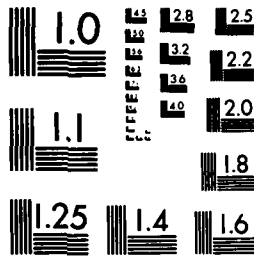
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MELBOURNE, VICTORIA

REPORT

MRL-R-1029

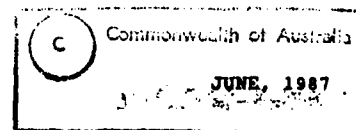
EFFICIENCY ASPECTS OF SHORT, LOW  
VELOCITY RAILGUNS

C.I. Sach

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MATERIALS RESEARCH LABORATORIES

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ABSTRACT

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The factors controlling the performance of short, low velocity railguns are studied and mathematical expressions defining the bounds of realizable efficiencies are obtained. The restrictions governing the validity of the expressions are discussed. The expressions are based on the proposition that during acceleration of the projectile in the railgun, the current falls almost linearly with time. Under the conditions discussed in this report it is shown that the maximum achievable efficiency is about 1.6%.

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# SYMBOLS

C	primary storage capacitance
$L_0$	intermediate storage inductance
$R_C$	circuit resistance in pre-crowbar phase
$R_1$	incremental resistance of crowbar switch
$R_2$	incremental resistance of plasma armature
$R_V$	circuit resistance during post-crowbar phase
$R_T$	total effective resistance post-crowbar
$R'$	resistance per unit length of railgun
$L'$	inductance per unit length of railgun
$V_1$	total voltage across crowbar switch
$V_2$	total voltage across plasma armature
$V_3$	total voltage across main switch
$E_1$	basic (constant) breakdown potential, crowbar
$E_2$	basic (constant) breakdown potential, armature
I	circuit current
$I_0$	initial maximum current
$I_E$	effective current $(E_1 + E_2)/R_T$
$I_M$	current in ideal railgun at shot-out
$I_1$	second current maximum in uncrowbarred gun
x	position co-ordinate of projectile in barrel
$l$	length of barrel
v	projectile velocity
$v_E$	projectile exit velocity at muzzle
$\tau$	circuit time constant $(L_0/R_T)$
T	time when current I reaches zero
t	time variable
$t_E$	time to shot-out from crowbarring

$f$	scaling factor related to railgun inductance parameter $L'$
$m_p$	mass of projectile
$m_A$	mass of plasma armature
$m$	combined mass of plasma armature and projectile ( $m_A + m_p$ )
$u$	dimensionless time parameter ( $t_E/T$ )
$w$	dimensionless railgun parameter
$E_p$	kinetic energy of projectile at muzzle
$g(u)$	defined function of $u$
$h(w)$	defined function of $w$
$\eta_T$	energy-transfer efficiency from capacitance to inductance energy store
$\eta_I$	overall efficiency of ideal railgun
$\eta$	overall efficiency of "practical" railgun
$\eta_M$	overall upper bound efficiency of "practical" railgun

## CONTENTS

	Page No.
1. INTRODUCTION	1
2. RAILGUN MODEL	2
2.1 General Description	2
2.2 Pre-crowbar Phase	2
2.3 Post-crowbar Phase	3
3. EVALUATION OF REPRESENTATIVE RAILGUN PARAMETERS	6
3.1 Plasma Armature Parameters ( $E_2$ , $R_2$ )	6
3.2 Combined Potential ( $E_1 + E_2$ ) and Resistance $R_T$	7
3.3 Crowbar Breakdown Potential $E_1$	7
3.4 Pre-crowbar Circuit Resistance $R_C$	8
4. CONVERSION EFFICIENCY	8
4.1 The Ideal Railgun	8
4.2 General Derivation - A More Realistic Model	9
4.3 Summary of Restrictions	12
4.4 Maximum Achievable Efficiency	14
4.5 Maximum Velocity Limit	14
5. DESIGN EQUATIONS FOR SMALL RAILGUNS	15
6. CONCLUSIONS	16
7. ACKNOWLEDGEMENTS	17
8. REFERENCES	18

EFFICIENCY ASPECTS OF SHORT, LOW  
VELOCITY RAILGUNS

1. INTRODUCTION

For some years MRL carried out laboratory studies on small railguns. The aim of that work was to understand the basic physical processes involved in railgun operation and for that purpose small experimental railguns served very well. The experimental railguns were not designed to meet any military requirements and hence the need for high conversion efficiency was relatively unimportant. In fact the railguns used were inefficient with typically less than 2% of the input energy being converted to kinetic energy of the projectile. Nevertheless the question of efficiency has always been addressed in MRL's work and in particular the way in which input energy is distributed throughout the railgun system has been investigated in the experimental studies.

The background and results of MRL work have been reported previously [1-13]. Railguns with a 6 x 8 mm bore (RAPID and ERGS) [4] as well as a 10 x 10 mm bore (RIPPER) were used. Theoretical aspects of efficiency of the RAPID railgun were considered by Kowalenko [11] in his study of a firing series codenamed RPIP.

The work presented here examines some aspects of the efficiency question, taking into account most of the effects likely to occur in a practical system. The work relates to what is loosely termed a short railgun, the essential criterion defining a short gun being established in the report. The key to the study is the derivation of a simple mathematical expression for the current flow in the railgun during firing. This allows some useful expressions relating to the efficiency to be obtained. It is emphasised that the railgun configuration being considered here represents a specific class having limited applications; it would not be used to meet the more usual requirements of high velocity and high efficiency.

In this report various efficiency parameters are introduced. The efficiency with which electrical energy, initially stored in a capacitance, is transferred to a secondary inductance store is designated as  $\eta_T$ . For



reference purposes the overall efficiency,  $\eta$ , of an ideal railgun is then derived. A general expression for the overall efficiency,  $\eta$ , of a more practical railgun is then obtained together with an estimate of its upper bound  $\eta_M$ . Overall efficiency refers to the ratio of the kinetic energy of the railgun projectile as it leaves the gun to the energy initially stored in the primary capacitance store.

## 2. RAILGUN MODEL

### 2.1 General Description

In this study the conventional railgun model as shown in Fig. 1 is considered. A storage capacitance  $C$  is initially charged to a potential  $V_0$ . During the pre-crowbar phase the main switch (a plasma-gap device or ignitron) is "closed", thus allowing the storage inductance  $L_0$  to be charged via the plasma armature established behind the projectile in the railgun. When the current reaches its maximum value  $I_0$  the crowbar switch isolates the capacitance from the inductance driving circuit. In practice the crowbarring is not achieved instantaneously and there is some minor ongoing interaction between the two circuits via the common crowbar path. It is emphasized that in this study the projectile is considered as initially at rest; the study does not cover injected railgun configurations [12].

### 2.2 Pre-crowbar Phase

The pre-crowbar phase starts with the closure of the main switch and consists of a short-duration energy transfer process in which the plasma armature is established and the inductance is fully charged to its maximum current. Although there is some movement of the projectile during this interval it will be considered negligible. The equivalent electrical circuit is as shown in Fig. 2. The voltage-drops  $V_2$  and  $V_3$  across the plasma armature and the main switch respectively are much smaller than the initial voltage  $V_0$  on the capacitance. Under these conditions the only significant loss during energy transfer is due to the circuit resistance  $R_c$ . It has been shown [11,12] that the energy necessary to establish the arcs is negligible in comparison with the other energy-dissipating processes. As shown in any standard circuit analysis text covering lightly-damped tuned circuits, the maximum current  $I_0$  built up in the inductance  $L_0$  is given by

$$I_0 = V_0 \sqrt{\frac{C}{L_0}} \exp \left\{ -\frac{R_c}{4} \sqrt{\frac{C}{L_0}} \right\} \quad (1)$$

provided  $R_c \sqrt{\frac{C}{L_0}} \ll 1$

This latter condition should be met for any well-designed system.

The efficiency of the energy transfer is defined as

$$\eta_T = \frac{\frac{1}{2} L_o I_o^2}{\frac{1}{2} C V_o^2}, \quad (2)$$

which, by the use of equation (1), becomes

$$\eta_T = \exp \left\{ -\frac{R_C}{2} \sqrt{\frac{C}{L_o}} \right\} \quad (3)$$

In general one could expect that a railgun designed to meet high efficiency specifications would aim to achieve  $\eta_T$  values in the range  $0.8 < \eta_T < 1$ . However values as low as 0.6 have been observed experimentally in MRL work where efficiency has not been a major consideration.

### 2.3 Post-crowbar Phase

The situation at some general time in the post-crowbar phase is shown in Fig. 3.  $V_1$  is the voltage across the crowbar switch and, as before  $V_2$  is the voltage across the plasma armature in the railgun. The projectile of mass  $m_p$ , together with the armature which is assumed to have a constant mass  $m_a$ , has moved a distance  $x$  along the barrel whose length is  $l$ . In accordance with usual railgun analyses, the railgun is characterised by the two parameters  $L'$  and  $R'$  which are, respectively, the railgun barrel inductance and resistance per unit length. The circuit resistance  $R_V$  accounts for all resistive effects apart from those associated with the railgun armature and the crowbar switch.

An examination of the records of MRL railgun firings shows that the behaviour of the two plasma regions can be represented electrically by constant breakdown potentials  $E_1$  and  $E_2$  in association with small incremental resistances  $R_1$  and  $R_2$

$$\begin{aligned} \text{i.e.} \quad V_1 &= E_1 + R_1 I \text{ and} \\ V_2 &= E_2 + R_2 I, \end{aligned} \quad (4)$$

where  $I$  is the railgun current.

This matter is discussed in Section 3 of the report. The electrical equivalent circuit can then be redrawn as shown in Fig. 4. (The symbols for voltage break-down diodes have been used in the equivalent circuits to emphasize the electrical behaviour being attributed to the plasmas in this study).

The circuit equation for Fig. 4 is

$$L_0 \frac{dI}{dt} + L'x \frac{dI}{dt} + L'I \frac{dx}{dt} + E_1 + E_2 + (R_v + R_1 + R_2 + R'x) I = 0 \quad (5)$$

The second term in equation (5) may be neglected in comparison with the first if  $L'x \ll L_0$ . As the largest value of  $x$  will be the barrel length,  $\ell$ , an appropriate condition is  $\ell L' \leq L_0/10$ . We are therefore limited in this study to railguns having an upper bound on length given by

$$\ell \leq \frac{1}{10} \frac{L_0}{L'} \quad (6)$$

The condition introduced above suggests that one possible way of classifying railgun systems for analysis purposes at least, is to use the system length parameter  $L_0/L'$ . Although this does not represent a physically measurable length as such, it may be used to classify a system as being short, medium or long according to whether  $\ell \ll L_0/L'$ ,  $\ell \sim L_0/L'$  or  $\ell \gg L_0/L'$  respectively. In the present study we are concerned only with railguns which are short in the sense indicated above.

Returning to equation (5) the third term can be neglected in comparison with the first; a sufficient but probably over-conservative condition being

$$L'I_0 v_E \leq \frac{1}{10} \left| \frac{dI}{dt} \right|_{\max}, \quad (7)$$

where  $v_E$  is the muzzle, or exit, velocity of the projectile. It will further be considered that the maximum resistance of the rails,  $R'\ell$ , is much smaller than all other resistances combined, i.e. the condition

$$R'\ell \ll R_v + R_1 + R_2 \quad (8)$$

is satisfied. This is the case with the RAPID railgun.

Under the above conditions, the electrical equivalent circuit reduces to Fig. 5 and the system equation becomes

$$L_0 \frac{dI}{dt} + (E_1 + E_2) + R_T I = 0, \quad (9)$$

where  $R_T$  represents all the circuit resistances. If we consider this second phase starting at  $t = 0$ , then the initial condition for equation (9) is  $I(0) = I_0$  where  $I_0$  is given by equation (1).

The solution to equation (9) is

$$I = (I_0 + I_E) \exp\left(-\frac{t}{\tau}\right) - I_E, \quad (10)$$

$$\text{where } I_E = \frac{E_1 + E_2}{R_T}, \quad (11)$$

$$\text{and } \tau = \frac{L_0}{R_T} \quad (12)$$

The current variation defined above can be expressed in the normalized form

$$\frac{I}{I_0} = \left(1 + \frac{I_E}{I_0}\right) \exp\left[-\frac{t}{T} \ln\left(1 + \frac{I_0}{I_E}\right)\right] - \frac{I_E}{I_0}, \quad (13)$$

where  $T$  is the time at which the current reaches, and then remains at, zero level.  $T$  is given by

$$T = \tau \ln\left(\frac{I_0 + I_E}{I_E}\right) \quad (14)$$

Equation (13) is plotted for  $I_E/I_0 = 1$  in Fig. 6A. For large values of  $I_E/I_0$ , i.e. where  $I_E/I_0 \rightarrow \infty$ , equation (13) reduces to

$$I = I_0 \left(1 - \frac{t}{T}\right), \quad 0 \leq t \leq T \quad (15)$$

It is demonstrated in Fig. 6B that this linear relationship is a good approximation to equation (13) for a wide range of  $I_E/I_0$  values, in particular for  $I_E/I_0 \geq 1$ . In the remainder of this report we will therefore regard  $I_E \geq I_0$ , i.e.

$$\frac{E_1 + E_2}{R_T} \geq I_0, \quad (16)$$

as a necessary and sufficient condition justifying the general use of the linear current relationship given by equation (15). The linear reduction of  $I$  with time as defined by equation (15) is clearly evident in the results of RAPID and RIPPER firings; in fact it was this unexpected linearity which led to the present analysis being undertaken. It should be noted that the analysis is valid only when  $t \leq T$  and while the projectile and plasma armature remain within the railgun. In fact in all MRL firings the projectile left the muzzle at some time  $t_E$  less than  $T$ . In such cases the analysis is only valid for  $t \leq t_E$ . In many experimental firings, though not all, the linear current variation is observed beyond  $t_E$  down to the point where the current approaches zero. An example of this behaviour is shown in Fig. 7.

If the conditions introduced in the derivation of equation (10) are examined, it is seen that they amount, in effect, to neglecting the presence of the railgun except for the plasma parameters  $E_2$  and  $R_2$ . This means that effects relating to  $R'I$ ,  $L'I (dx/dt)$  and  $L'x (dI/dt)$  are considered negligible. These are the terms which arise from railgun action and cause the difference between the breech voltage  $V_B$  and the muzzle voltage  $V_M$  (defined in Figs. 3 and 4). Representative data from a RAPID firing are shown in Fig. 8. Here the two voltages are seen to be essentially the same, i.e. the railgun, as far as electrical modelling purposes are concerned, can be considered to have the plasma armature located at the breech.

The important conclusion which follows directly is that the projectile's position and velocity within the railgun have negligible effect on the electrical driving circuit, i.e. the electrical circuit aspects of the gun system can be considered to be independent of mechanical aspects such as projectile position and velocity. The converse is not true.

### 3. EVALUATION OF REPRESENTATIVE RAILGUN PARAMETERS

#### **3.1 Plasma Armature Parameters ( $E_2$ , $R_2$ )**

A typical breech voltage record is shown in Fig. 9 and its corresponding current-voltage relationship in Fig. 10. (This result was taken using a RAPID gun in a series known as MRAP designed to test magnetic flux sensors.) Fig. 9 shows the characteristic slow run-down with the superimposed fluctuations up to shot-out and then the characteristic jump and more rapid run down until the arc is finally extinguished at the muzzle as the current reaches zero. Ignoring the randomlike fluctuations in Fig. 10, which are probably due to short-duration dynamic processes within the plasma volume, it is possible to represent the breech voltage  $V_B$  for the in-barrel phase as

$$V_B \approx 150 + 0.7 \times 10^{-3} I$$

from which, by equating  $V_B$  with  $V_2$  and recalling that  $V_2 = E_2 + R_2 I$  as discussed in section 2.3, we obtain  $E_2 = 150$  V and  $R_2 = 0.7 \times 10^{-3} \Omega$ . As a matter of interest the breech voltage-current curve for the muzzle arc is also

shown in Fig. 10. This shows linear behaviour, also tending to  $V_B = 150$  V, as the current approaches zero.

### 3.2 Combined Potential ( $E_1 + E_2$ ) and Resistance $R_T$

A curve-fitting procedure was used to estimate ( $E_1 + E_2$ ) and  $R_T$ . This involved plotting current-time data from experimental firings in the form ( $I/I_0$ ) on a logarithmic scale versus time, as proposed by Kowalenko [11], and then comparing this with the theoretical curve (derived from equations (10), (11) and (12)),

$$\text{i.e.} \quad \frac{I}{I_0} = 1 - \frac{E_1 + E_2}{L_0} t \left( 1 - \frac{R_T}{2L_0} t \right) \quad (17)$$

which was evaluated for assumed values of ( $E_1 + E_2$ ) and  $R_T$ . This essentially trial-and-error procedure leads to unique values of ( $E_1 + E_2$ ) and  $R_T$  for best fit.

A representative fit is shown in Fig. 11 where it is found that ( $E_1 + E_2$ ) = 300 V and  $R_T = 2.7 \times 10^{-3} \Omega$ . The error in parameter estimation is probably about 10%.

### 3.3 Crowbar Breakdown Potential $E_1$

The first method used to evaluate  $E_1$  involved direct measurement of the voltage across the crowbar switch during a railgun firing. The difficulty with this method is that when the main switch is closed during the pre-crowbar phase, the full capacitor voltage  $V_0$ , which is typically 6 kV, appears across the crowbar switch. Experimental results show that during operation of the crowbar the potential reverses by approximately 1000 V before settling towards a value of about 200 V. In order to obtain these data a voltage divider (1000 times) was used to give the typical record shown in Fig. 12. The errors in the measurement are such that the value of  $V_1 \approx 200$  V can only be regarded as approximate. The data are not sufficiently accurate to estimate  $E_1$  or  $R_1$  reliably.

A second method for determining  $E_1$  was based on monitoring the current  $I$  during the operation of a railgun in which no projectile or foil was present but where a low resistance strap was placed across the muzzle. This arrangement effectively reduced the system to that shown in Fig. 13 during the post-crowbar phase. Although the resistance  $R$  will not correspond to  $R_T$  in the railgun circuit, the value  $E_1$  should be the same as in a normal firing. The theoretical current relationship for the circuit proposed in Fig. 13 is

$$I = \left( I_0 + \frac{E_1}{R} \right) \exp \left( - \frac{tR}{L_0} \right) - \frac{E_1}{R} \quad (18)$$

and this was compared with experimental data. The parameter values best matching the experimental curve shown in Fig. 14 are  $E_1 = 145$  V and  $R = 1.67$  m $\Omega$ .

### 3.4 Pre-crowbar Circuit Resistance $R_C$

The most direct method for estimating  $R_C$  involves the use of equation (1) with measured values of  $V_0$ ,  $I_0$ ,  $C$  and  $L_0$ . Kowalenko [11] has drawn attention to some uncertainty in the calibration values for the current records obtained in the RAPID and RIPPER firings at MRL. An alternative method, using the results of a firing when the crowbar switch failed to operate, was therefore used. The waveform record, clipped on the negative-swing by limitations of the data-capture equipment, is shown in Fig. 15. The occurrence of the two successive current maxima  $I_0$  and  $I_1$ , with the projectile still in bore, allows  $R_C$  to be estimated without the need for accurate current calibration. It can be readily shown that

$$R_C = -\frac{1}{\pi} \sqrt{\frac{L_0}{C}} \ln \left( \frac{I_1}{I_0} \right) \quad (19)$$

Substituting the following values which are representative of the RAPID railgun ( $L_0 = 6.3 \times 10^{-6}$  H,  $C = 1600$   $\mu$ F and  $I_1/I_0 = 0.71$ ), equation (19) gives  $R_C$  as  $7 \times 10^{-3}$   $\Omega$ . The transfer efficiency  $\eta_T$  is then found from equation (3) to be 89%.

## 4. CONVERSION EFFICIENCY

### 4.1 The Ideal Railgun

An inductance  $L_0$  supplying current to a railgun with inductance per unit length  $L'$  and barrel length  $\ell$  as shown in Fig. 16 is considered. The inductance, the rails and the armature are assumed to have zero resistance. Because the sum of the voltage drops around the circuit is zero, the circuit equation is

$$\frac{d}{dt} (L_0 I + L' \ell I) = 0 \quad (20)$$

Integrating equation (20) between shot-start and shot-out leads to

$$(L_0 + L' \ell) I_M = L_0 I_0$$

where  $I_M$  is the railgun current at shot-out. It follows that

$$I_M = I_0 \frac{L_0}{L_0 + L'\ell} \quad (21)$$

Now if it is assumed that the reduction in stored magnetic energy corresponds to the increase in the kinetic energy  $E_p$  of the projectile, then

$$E_p = \frac{1}{2} L_0 I_0^2 - \frac{1}{2} (L_0 + L'\ell) I_M^2$$

i.e.

$$E_p = \frac{1}{2} \frac{L_0 L'\ell}{L_0 + L'\ell} I_0^2 \quad (22)$$

The conversion efficiency  $\eta_I$  is then given by

$$\eta_I = \frac{E_p}{\frac{1}{2} L_0 I_0^2} \quad (23)$$

which, with equation (22), yields

$$\eta_I = \frac{L'\ell}{L_0 + L'\ell} \quad (24)$$

This expression is quite general and, as would be expected,  $\eta_I$  approaches unity for  $L'\ell \gg L_0$ . For the case of short railguns, as discussed in Section 2.3 where  $\ell \ll L_0/L'$ , equation (24) becomes

$$\eta_I = \frac{L'\ell}{L_0} \quad (25)$$

As an example, for a railgun like RAPID, with  $\ell = 0.5$  m,  $L_0 = 6.3$   $\mu$ H and  $L' \approx 0.5$   $\mu$ H/m the value of  $\eta_I$  is 0.04, i.e. the highest possible efficiency is 4%. It should be noted that this result is independent of the projectile mass, the current-time waveform and projectile motion providing only that equation (21) applies.

#### 4.2 General Derivation - A More Realistic Model

In this section we consider the railgun discussed in Section 2 and in particular we will consider that the railgun current decreases linearly with time according to equation (15). The usual equation of motion is taken, i.e.

$$m \frac{dv}{dt} = \frac{1}{2} f L' I^2, \quad (26)$$



where  $v = \frac{dx}{dt}$  is the projectile velocity, and

$$m = m_A + m_p$$

where  $m_p$  is the mass of the projectile, and  $m_A$  is the mass of the plasma armature and  $f$  is a factor ( $f < 1$ ) introduced by Sadedin (13) to allow for the effect of leakage inductance. This results in a difference between the circuit magnetic flux and the effective flux contributing to the force acting on the plasma. The use of equation (26) implies that gas forces ahead of the projectile and friction forces are negligible, and that  $m_A$  is a constant. More specifically it means either that no material is picked up and expelled during armature motion or, if it is, it is ejected at the same mass rate as it is picked up and, additionally, is ejected at zero velocity relative to the armature.

Using equation (15) for the current  $I$ , integration of equation (26), leads to

$$mv = \frac{1}{6} f L' I_0^2 T \left[ \left(\frac{t}{T}\right)^3 - 3\left(\frac{t}{T}\right)^2 + 3\left(\frac{t}{T}\right) \right] \quad (27)$$

which holds for  $0 \leq t \leq T$  but only whilst the projectile remains in the barrel, i.e.  $x \leq \ell$ .

Using equation (27), the shot-out condition is

$$mv_E = \frac{1}{6} f L' I_0^2 [u^3 - 3u^2 + 3u], \quad 0 \leq u \leq 1 \quad (28)$$

$$\text{where} \quad u = \frac{t_E}{T}, \quad (29)$$

$t_E$  is the time of projectile exit and

$v_E$  is the exit velocity.

The condition  $0 \leq u \leq 1$  in equation (28) means that the expression is only valid where the projectile reaches the muzzle before, or just as, the current reaches zero. The condition  $u > 1$  (i.e.  $t_E > T$ ) means the barrel is unnecessarily long because the current is zero for  $t > T$ .

The general expression defined by equation (27) can also be integrated to provide a relationship between barrel length  $\ell$  and  $u$  as follows:

$$m\ell = \frac{1}{24} f L' I_0^2 T^2 [u^4 - 4u^3 + 6u^2] \quad (30)$$

The kinetic energy of the projectile at exit is given by

$$E_p = \frac{1}{2} m_p v_E^2$$

i.e.

$$E_p = \frac{1}{2} \frac{m_p \ell}{m_A + m_p} \frac{(mv_E)^2}{m \ell} \quad (31)$$

Using  $mv_E$  and  $m \ell$  from equation (28) and (30) respectively then gives

$$E_p = \frac{1}{2} L' I_o^2 f \frac{m_p}{m_A + m_p} g(u), \quad (32)$$

where

$$g(u) = \frac{2[(u-1)^3 + 1]^2}{3u^2 [u^2 - 4u + 6]} \quad (33)$$

The efficiency of the railgun  $\eta$  which is the ratio of the exit kinetic energy to the original stored capacitor energy, is given by

$$\eta = \eta_T \frac{E_p}{\frac{1}{2} L_o I_o^2} \quad (34)$$

where  $\eta_T$  is the energy transfer efficiency discussed in Section 2. Using the expression for  $E_p$  obtained above

$$\eta = \eta_T \frac{m_p}{m_p + m_A} f \frac{L' \ell}{L_o} g(u) \quad (35)$$

The last expression is not directly useful as it stands. A more useful approach is to express efficiency in terms of a parameter

$$w = \frac{24(m_A + m_p) \ell}{f L' I_o^2 T^2} \quad (36)$$

which may also be expressed in terms of basic parameters in the form

$$w = \frac{24(m_A + m_P) \ell R_T^2}{f L' I_0^2 L_0^2 \ln^2 \left[ 1 + \frac{I_0 R_T}{E_1 + E_2} \right]} \quad (37)$$

The relationship between  $u$  and  $w$  follows from equation (30) as

$$u^2 (u^2 - 4u + 6) = w \quad (38)$$

This function is plotted in Fig. 17 where it is clear that the restriction  $0 \leq u \leq 1$  leads to the corresponding restriction  $0 \leq w \leq 3$ . A function  $h(w)$  can then be generated by combining equations (33) and (38) under the equality  $g(u) = h(w)$  so that efficiency can be expressed in the form

$$\eta = \eta_T \frac{m_P}{m_P + m_A} f \frac{L' \ell}{L_0} h(w) \quad (39)$$

The function  $h(w)$  is shown in Fig. 18.

Given the parameters of a short railgun, equations (37) and (39) can be used to evaluate the expected efficiency. If the value of  $w$  exceeds three it means the actual barrel length  $\ell$  is longer than necessary. Its effective length will then be that value of  $\ell$  which makes  $w = 3$ .

#### 4.3 Summary of Restrictions

In deriving the efficiency expressions in the previous sections a number of conditions were imposed. These are not physical restrictions on railgun parameters as such but are conditions relating to the notion of short railguns as used in this report and relate to the validity of the mathematical expressions which have been derived. The first somewhat arbitrary, conditions are

$$\ell \leq \frac{1}{10} \frac{L_0}{L'} \quad \text{from equation (10),}$$

$$R' \ell \ll R_1 + R_2 + R_V \quad \text{from equation (8),}$$

$$\text{and} \quad \frac{E_1 + E_2}{R_T} \geq I_0 \quad \text{from equation (16).}$$

A third condition discussed in Section 4.2 is that the parameter  $w$  has a maximum value of 3. (It will be shown below that there is also a modified minimum permitted value greater than zero).

A fourth condition was introduced as equation (7), viz

$$L' I_0 v_E \leq \frac{1}{10} L_0 \left| \frac{dI}{dt} \right|_{\max}$$

This will now be examined more fully. From equation (15) it follows that  $dI/dt$  is a constant ( $-I_0/T$ ). Substituting this in the above equation gives

$$v_E \leq \frac{1}{10T} \frac{L_0}{L'} \quad (40)$$

This value is, in effect, the upper limit on allowable muzzle velocity for which the present model applies. Now from equation (28) the maximum possible exit velocity, obtained for  $u = 1$ , is

$$v_{E_{\max}} = \frac{f L' I_0^2 T}{6 (m_A + m_p)}$$

It then follows that the condition to be met is

$$\frac{f L' I_0^2 T}{6 (m_A + m_p)} \leq \frac{1}{10T} \frac{L_0}{L'} \quad (41)$$

which can be rearranged to give

$$w \geq \frac{40 \ell L'}{L_0} \quad (42)$$

where  $w$  is as defined previously. But it was also stipulated earlier that  $w \leq 3$ . Hence the maximum allowable value of  $\ell L'/L_0$  for which this present analysis can be considered valid is

$$\frac{\ell L'}{L_0} = \frac{3}{40} \quad (43)$$

This condition is slightly more severe than  $\ell L'/L_0 \leq 1/10$  so it will now be used as the defining condition for a short railgun. For practical purposes of course the difference is of little consequence. An important conclusion is that the expression for efficiency  $\eta$  given in equation (39) is valid only in the range

$$40 \frac{\ell L'}{L_0} \leq w \leq 3 \quad (44)$$

As an example of the use of the previous equations, consider a railgun with the following characteristics:  $m_p = 10^{-3}$  kg,  $\ell = 0.4$  m,  $R_T = 3 \times 10^{-3}$   $\Omega$ ,  $I_0 = 10^5$  A,  $E_1 + E_2 = 300$  V,  $f = 0.6$ ,  $L_0 = 5 \times 10^{-6}$  H,  $L' = 0.5 \times 10^{-6}$  H/m and  $\eta_T = 0.9$ . It will be assumed that  $m_A = 0$ . The value of  $w$  from equation (37) is  $w = 2.3$ . Because  $40 \ell L' / L_0 = 1.6$ , the condition in equation (44) is met and hence the expressions given in the report can be used to determine  $h(w)$  and  $\eta$ . From Fig. 17  $h(2.3) = 0.28$ . From equation (39) we then obtain  $\eta = 0.6\%$ .

#### 4.4 Maximum Achievable Efficiency

In the previous sections, mathematical expressions were obtained which allow the upper bound on efficiency of a short railgun ( $\ell L' / L_0 \leq 3/40$ ) to be estimated. However it is evident from equation (39) that, for a given value of  $\ell L' / L_0$  it is necessary that  $h(w)$  be made as large as possible if maximum efficiency is to be achieved. From Fig. 18 it is seen that this requires  $w$  to be minimized. But from equation (42) we know that this minimum value must be  $40 \ell L' / L_0$ . Using these values we obtain an expression for maximum efficiency  $\eta_M$  using equation (39), which can be expressed as follows:

$$\frac{\eta_M}{\eta_T \left( \frac{m_p}{m_p + m_A} \right)} = \frac{L' \ell}{L_0} h\left(40 \frac{\ell L'}{L_0}\right) \quad (45)$$

The function is plotted in Fig. 19. It is seen that the curve lies between the straight lines with slope 1 (corresponding to the ideal railgun) and slope 2/9. The important conclusion is that the maximum achievable efficiency is  $2/9(\ell L' / L_0)$  as  $\ell L' / L_0$  approaches  $3/40$ , i.e. the maximum efficiency is 1.6%. For interest, the probable operating regime of the MRL RAPID railgun is shown in Fig. 19. Typical measured values of efficiency are 0.7% for RAPID [4] and 1.1 to 2.1% for ERGS I-M [6]. It should be noted that the experimental values will include gas-pressure propulsion effects which can be significant in short railguns [13].

#### 4.5 Maximum Velocity Limit

An upper limit on the muzzle velocity for  $v_E$  for which the present model can be considered valid is given by equation (40) which, in conjunction with the expression for  $T$  derived at equation (14) earlier, gives

$$v_E \leq \frac{1}{10 \ln \left[ 1 + \frac{I_0 R_T}{E_1 + E_2} \right]} \left( \frac{R_T}{L'} \right) \quad (46)$$

The behaviour of this function is shown in Fig. 20 for circuit resistance values  $R_T$  in the range 0 to 5 m $\Omega$  and railgun currents  $I_0$  in the range 25 to 200 kA. Values of  $E_1 + E_2 = 300$  and  $L' = 0.4$   $\mu$ H/m were used for all cases.

It is evident that even at the low peak current of 25 kA, the maximum velocity is only 3 km/s. This value lies at the low velocity end of the range of possible applications for which railguns are usually proposed. Further, it must be noted that low currents will, for barrels of practical lengths (e.g. 5 m), be incapable of propelling masses much above 0.1 g, i.e. the practical usefulness at low currents is very restricted. It is apparent therefore that the model and results developed here must be considered as appropriate only for low velocity guns.

##### 5. DESIGN EQUATIONS FOR SMALL RAILGUNS

To complete this study a set of design equations will be obtained. Once again it must be emphasized that the results are only applicable to a restricted class of railgun applications where low velocities and low efficiencies are acceptable. It will be assumed that values of  $m_p$  and  $v_E$  have been specified and that a railgun giving maximum, albeit low, efficiency is to be designed. It will be taken that the values of  $f$ ,  $(E_1 + E_2)$  and  $L'$  are known for the railgun configuration proposed. The values of  $L_0$ ,  $\ell$ ,  $I_0$  and  $R_T$  are to be calculated. It will be assumed for convenience that  $m_A \ll m_p$ .

We know that maximum efficiency is achieved when  $w = 3$  or, equivalently,  $u = 1$ . If  $u = 1$  is substituted in equations (28) and (30) we obtain

$$m_p v_E = \frac{1}{6} f L' I_0^2 T \quad (47)$$

$$\text{and} \quad m_p \ell = \frac{1}{8} f L' I_0^2 T^2 \quad (48)$$

Combining (47) and (48) gives

$$\ell = \frac{9}{2} \frac{m_p v_E^2}{f L' I_0^2} \quad (49)$$

and then, using equation (43), we obtain

$$L_0 = 60 \frac{m_p v_E^2}{f I_0^2} \quad (50)$$

This means that  $\ell$  and  $L_0$  are known once  $I_0$  is determined. A value for  $I_0$  can be evaluated as follows. From equation (47)

$$T = \frac{6 m v_E}{f L' I_o^2}$$

Combining this with an expression for T obtained from equation (12) and (14) we obtain

$$\frac{L_o}{R_T} \ln \left[ \frac{I_o + \frac{E_1 + E_2}{R_T}}{\frac{E_1 + E_2}{R_T}} \right] = \frac{6 m v_E}{f L' I_o^2} \quad (51)$$

Following the discussion in section 2.3 the convenient, though somewhat arbitrary design stipulation

$$R_T = \frac{E_1 + E_2}{I_o} \quad (52)$$

is introduced, equations (50) and (51) then providing the following expression for  $I_o$ ,

$$I_o = \frac{1}{10 \ln 2} \frac{E_1 + E_2}{L' v_E} \quad (53)$$

Equations (53), (52), (50) and (49), taken in turn, provide the required values of  $I_o$ ,  $R_T$ ,  $L_o$  and  $\ell$ .

As an example, suppose  $m_p = 10^{-3}$  kg,  $v_E = 800$  m/s,  $f = 0.6$ ,  $\eta_T = 0.8$ ,  $L' = 0.5 \times 10^{-6}$  H/m and  $(E_1 + E_2) = 300$  V. Successive use of the above equations then gives  $I_o = 108$  kA,  $R_T = 2.8$  m $\Omega$ ,  $L_o = 5.5$   $\mu$ H and  $\ell = 0.8$  m. The efficiency  $\eta_M$  is then 1.3%.

## 6. CONCLUSIONS

Expressions have been obtained for the current flow in, and efficiencies available from, small, low velocity railguns of the type used for experimental work at MRL. It is shown that in the context of this report the condition defining a short railgun is  $\ell \leq 3/40 L_o/L'$ . The maximum efficiency is achieved when the current drive reaches zero simultaneously with the projectile reaching the muzzle of the gun. Under these conditions the maximum efficiency is 2/9 of that for an ideal short rail gun whose efficiency is  $\ell L'/L_o$ , i.e. the best achievable efficiency is 1.6% when  $\ell L'/L_o = 3/40$ .

The work has shown that the railgun current is controlled predominantly by the two constant voltage breakdown potentials  $E_1$  and  $E_2$  in

the plasma armature and in the (plasma) crowbar switch respectively and by the storage inductance  $L_0$  and the total circuit resistance  $R_T$ . For typical small MRL railguns, the current decreases linearly with time, independent of projectile motion in the railgun. This effective decoupling of the electrical from the mechanical effects has enabled a simple analysis of the projectile behaviour to be carried out. For the configurations considered here, the efficiencies are very low, with approximately 98% of the energy which is initially transferred to the drive inductance being dissipated in the two plasma arcs and the circuit loss resistances.

The fundamental electrode potentials  $E_1$  and  $E_2$  were shown to be about 150 V. No opportunity has been available to study the factors affecting these values in other railgun configurations. From the RPIP firings it would appear that the value of  $E_2$  is independent of the type and mass of metal foil used to establish the plasma armature.

#### 7. ACKNOWLEDGEMENTS

The author wishes to acknowledge the efforts of the many officers who have contributed to the experimental and theoretical work on railguns at MRL and whose results have been used in this report. The role played by Dr Kowalenko in prompting the study and the valuable assistance and guidance provided by Mr D. Sadedin and Dr D. Richardson in preparing this report is acknowledged.



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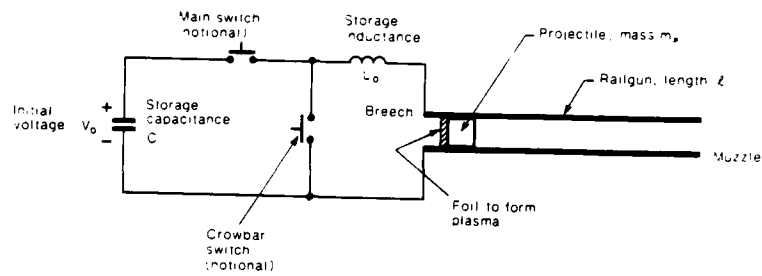


FIGURE 1 Railgun model.

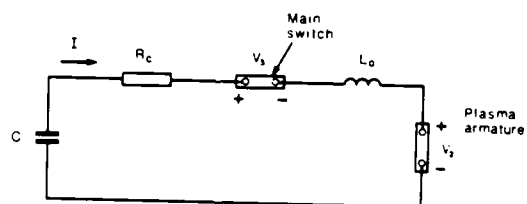


FIGURE 2 Pre-crowbar equivalent circuit.

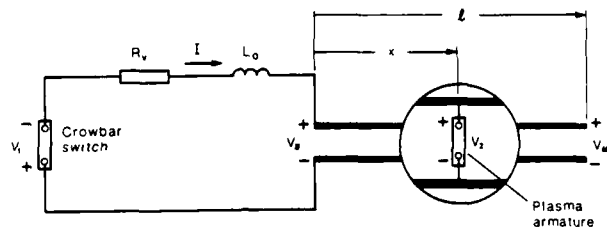


FIGURE 3 Post-crowbar equivalent circuit.

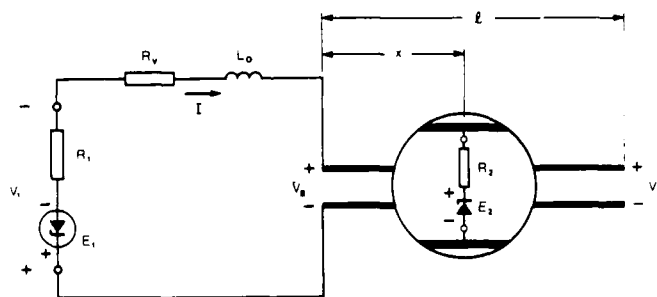


FIGURE 4 Post-crowbar equivalent circuit.

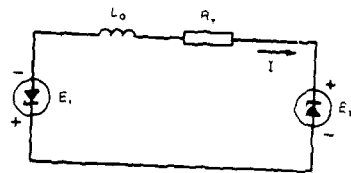


FIGURE 5 Simplified equivalent circuit.

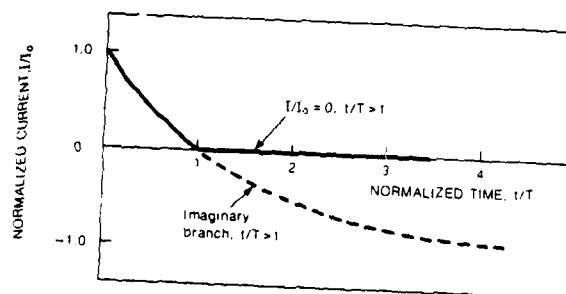


FIGURE 6A Current variation for  $I_E/I_0 = 1$ .

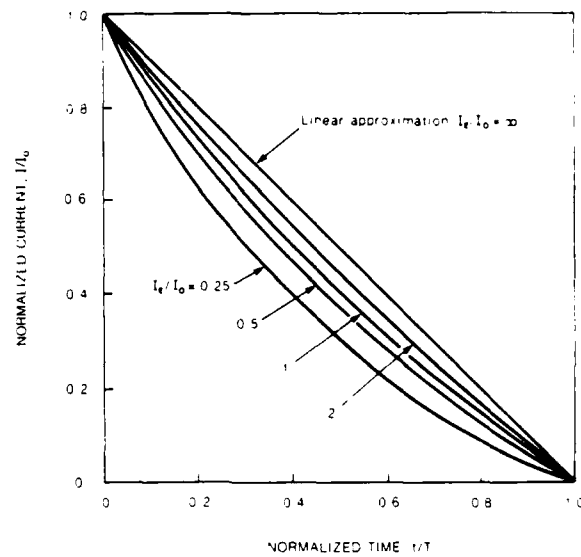


FIGURE 6B Current variation for different  $I_E/I_0$  ratios.

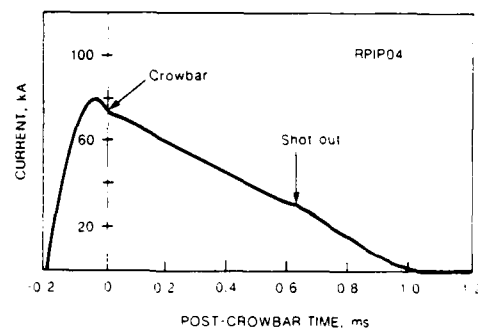


FIGURE 7 Experimental current record.

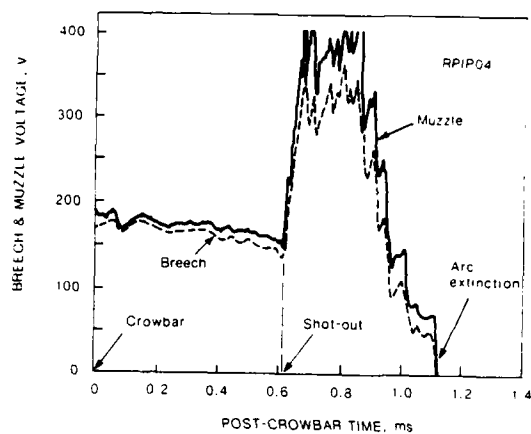


FIGURE 8 Experimental breech and muzzle voltage records.

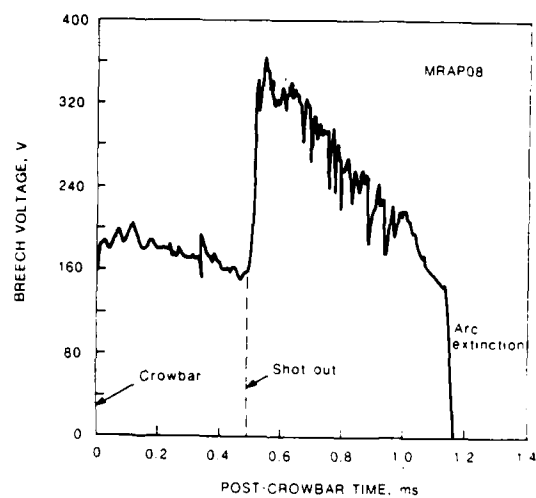


FIGURE 9 Post-crowbar breech voltage.

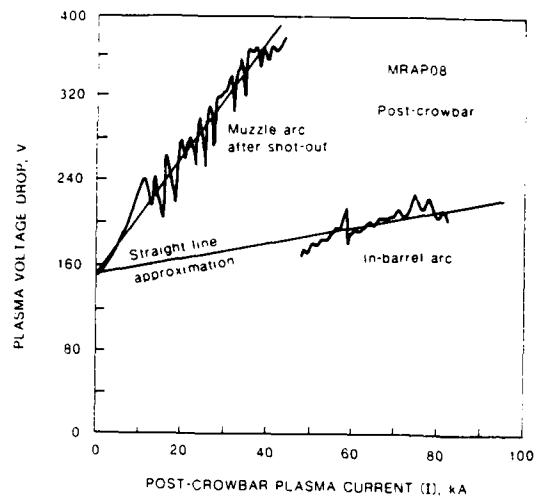


FIGURE 10 Plasma voltage-current relationships.

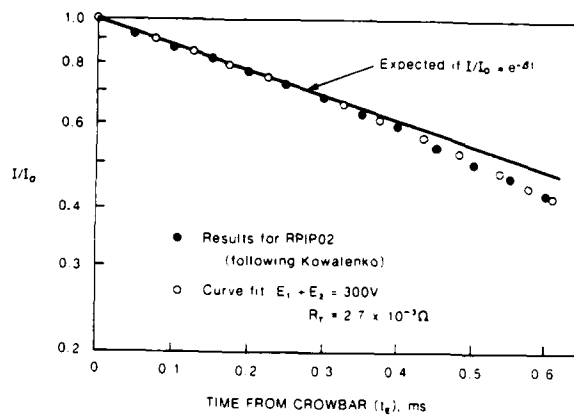


FIGURE 11 Parameter estimation by curve fitting.

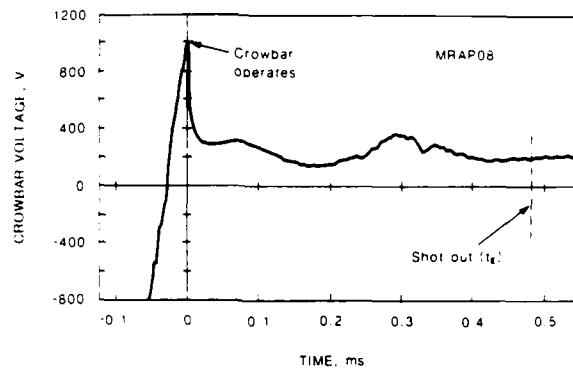


FIGURE 12 Crowbar switch voltage.

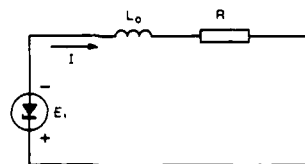


FIGURE 13 Equivalent circuit for low resistance muzzle strap with no projectile or plasma present.



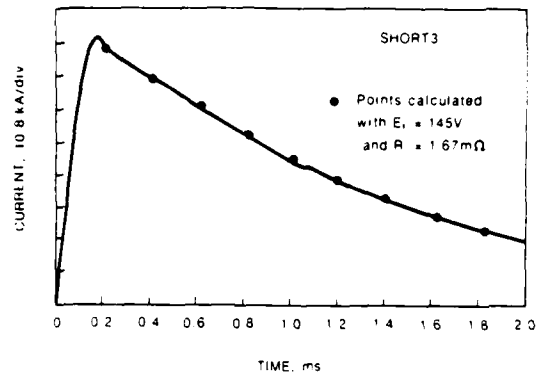


FIGURE 14 Experimental current record, no projectile and short-circuited muzzle.

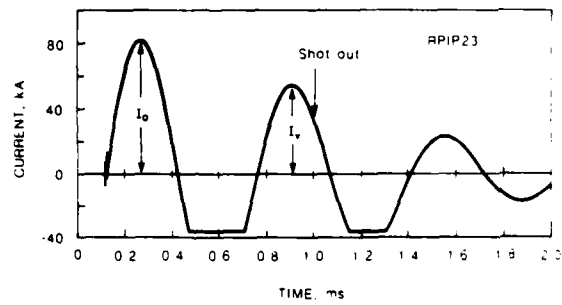


FIGURE 15 Experimental gun current with no crowbar switch.

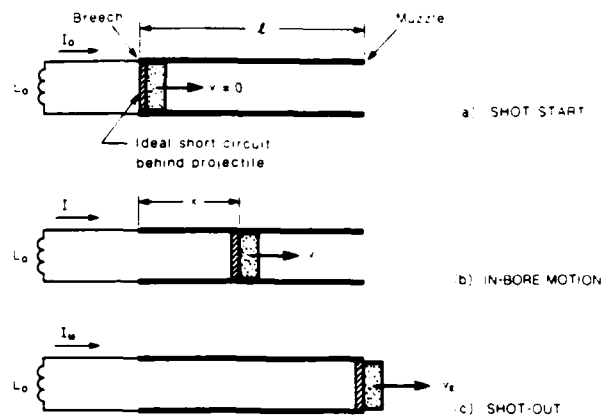


FIGURE 16 Stages in ideal railgun operation.

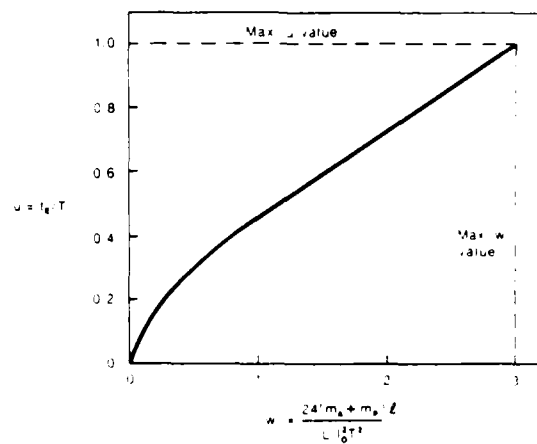


FIGURE 17 Relationship between  $u$  and  $w$ .

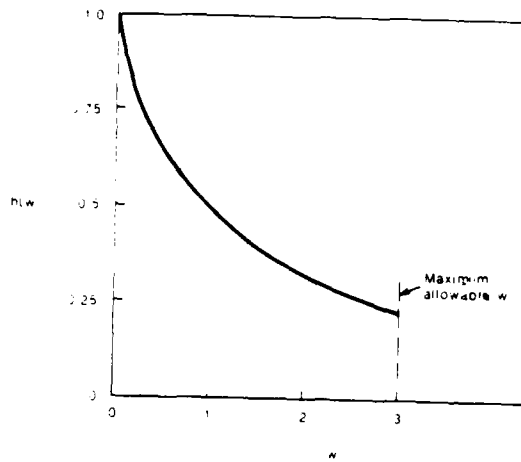


FIGURE 18 Plot of function  $h(w)$ .

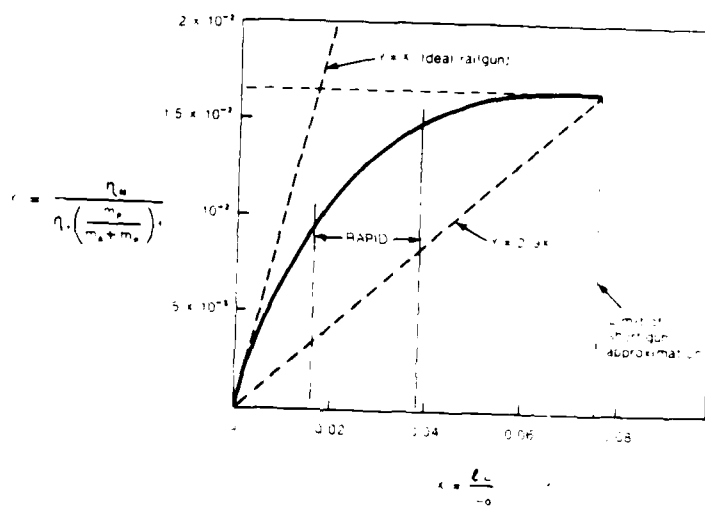


FIGURE 19 Maximum efficiency curve.

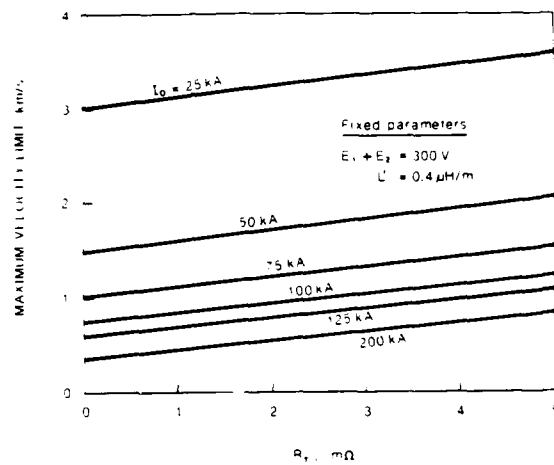


FIGURE 20 Maximum velocity limits on railgun model.

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ABSTRACT

The factors controlling the performance of short, low velocity railguns are studied and mathematical expressions defining the bounds of realizable efficiencies are obtained. The restrictions governing the validity of the expressions are discussed. The expressions are based on the proposition that during acceleration of the projectile in the railgun, the current falls almost linearly with time. Under the conditions discussed in this report it is shown that the maximum achievable efficiency is about 1.6%.

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